# Bio-calculus: Its Concept and Molecular Interaction 

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#### Abstract

The way for expressing biological systems is a key element of usability. Expressions used in the biological society and those in the computer science society have their own merits. But they are too different for one society to utilize the expressions of the other society. In this paper, we design the bio-calculus that attempts to bridge this gap. We provide syntax which is similar to conventional expressions in biology and at the same time specifies information needed for simulation analysis. The information and mathematical background of bio-calculus is what is desired for the field of computer science. We show the practicality of bio-calculus by describing and simulating some molecular interactions with bio-calculus.


## 1 Introduction

Recent progress of molecular biology and genomics is rapidly producing enormous amounts of experimental results. It is becoming almost impossible to formulate any hypotheses that are consistent with all these results depending solely on the human power. To overcome this situation, technology of computer science is strongly required.

There is a big gap between biology and computer science when they communicate to investigate biological phenomena. One of the problems is the difference between their expression systems. The expressions of biology clarify the components (e.g. gene, protein, and cells). Their transition and interaction (e.g. gene activation, protein phosphorylation and cell division) are diagramed. In many cases, mathematical models for these processes are not defined in biology. Thus, expressions of biology lack information essential for simulation analysis (e.g. the velocity of a process, the quantity of a component) and do not have background of mathematics (e.g. calculus and probabilistic arguments).

Expressions of computer science are designed to include information essential for simulation since simulation is one of the main analytical methods in this field. The expressions also have mathematical background since system analysis is another important method. Several expression systems have been proposed for expressing biological phenomena (e.g. differential equation system [5, 7], stochastic method [4], and stochastic differential equation system [1]). In these systems, components of biological systems are mingled with mathematical symbolism. Transition and interaction of the components are far less visualized than in the diagramed expressions of biology. This is a critical weak point for biology since the components and their transition/communication are the main targets of biology.

To make a bridge between biology and computer science, we have started to construct bio-calculus. The bio-calculus is an expression system that i) clarifies components of biological systems, ii) diagrams
their transition and communication, iii) includes information essential for simulation, and iv) possesses mathematical background. In this paper, we present this concept of bio-calculus. Then we define bio-calculus for molecular interaction. Finally, we discuss five major advantages of bio-calculus by expressing simple examples of molecular interaction.


Figure 1: Problems of expression among biology and computer science. Biocalculus solves those problems.

## 2 Bio-calculus

### 2.1 Requirements for bio-calculus

The expression of bio-calculus should satisfy the following requirements:
Req. 1 The expression clarifies components of biological systems.
Req. 2 The expression diagrams transition and communication of the components.
Req. 3 The expression includes information essential for simulation analysis.
Req. 4 The expression possesses background of mathematics.

### 2.2 Multi-semantics system for bio-calculus


(b) Multi-semantics system(for bio-calculus)


Figure 2: (a) A general calculus. (b) Multi-semantics system (for bio-calculus).

The distinguished feature of bio-calculus is multi-semantics.
To satisfy Req. 1 and Req. 2, the expression of bio-calculus is required to be quite similar to that of biology. Thus, we decided that in this calculus, each biological phenomenon is defined with the unique expression similar to that of biology. For Req. 3, we investigated several phenomena tring to define a single absolute simulation model for each phenominon. We then found that it is almost impossible to define such model for at least two reasons. Firstly, almost all phenomena are far from being completely understood. Thus, any explanation of a phenomenon will change. Secondly, there exist a huge number
of parameters - e.g. variety of molecule, pH and temperature. In many cases, some parameters are omitted for approximation. However, those parameters vary depending on what target is focused on. Accordingly, to fulfill Req. 3, each biological phenomenon should be defined by a unique expression with several explanations. This expression-explanation relationship is what multi-semantics system realizes in bio-calculus. To fulfill Req. 4, we define bio-calculus mathematically.

Bio-calculus is a calculus of multi-semantics system. Generally, a calculus consists of a syntax and a semantics corresponding to the syntax (Fig. 2(a)). Syntax is "the grammatical arrangement of words, showing their connection and relation". Semantics is "the evaluative rules of the language that is ruled by the syntax". In multi-semantics system, a syntax allows various kinds of semantics.

Bio-calculus consists of bio-syntax and bio-semantics-set. Bio-syntax is the syntax of this calculus, whose arrangement is designed to be quite similar to diagramed expressions of biology. Bio-semantics-set is a set of various kinds of bio-semantics. One of these bio-semantics is selected and applied to a bio-syntax. Thus, the semantics of a bio-syntax varies depending on the selected biosyntax. This syntax-semantics relationship is the special feature of bio-calculus. The relationship is quite analogous to the expression-explanation relationship in expressions for biological phenomena. Thus, it enables the calculus to fulfill Req. 1-4. We named a kind of system realizing this syntaxsemantics relationship multi-semantics system. We believe that this system is also quite useful for expressing other objects existing in the real world.

Selection of bio-semantics influences some parts of bio-syntax. So bio-syntax is divided into three parts, ID-independent-bio-syntax, semantic-ID, and ID-dependent-bio-syntax. ID-Independent-biosyntax is the part of bio-syntax that is never influenced by selection of semantic-ID. This part contains template-rule's, which represent components of biological systems, and their transition and communication. The syntax of template-rule is designed to be quite similar to the expressions of biology. Thus, biological expressions for target phenomena are easily transported to the calculus and become available for computational analysis. The result of computational analysis under the calculus is quite familiar with biology. A semantic-ID specifies a bio-semantics for a template-rule. This is the special term that realizes multi-semantics system and makes the syntax of template-rule resemble the expressions of biology. ID-dependent-bio-syntax is the remaining part of bio-syntax. This part consists of constants for calculation. Contents of this part vary depending on the selection of semantic-ID.

Hence, all these parts are organized in this way and they function as a multi-semantics system.

### 2.3 Advantages of bio-calculus

Bio-calculus has many advantages. Some of them are directly related to Req. 1-4 and others are benefits of the multi-semantics system. In this section, we focus on five advantage that will strongly contribute to biology. The first two advantages are directly related to Req. 1-4. The others are the benefits of the multi-semantics system. In Section 4, each advantage is discussed with a simple example of molecular interaction.

## Advantages

1. A biological expression is quite similar to a template-rule of bio-calculus. Thus the expression can be easily transported to bio-calculus. Then, the expression of bio-calculus is ready to be analyzed computationally.
2. An expression of bio-calculus includes information essential for computational analysis. Thus, the expression can be analyzed computationally. Results of computational analysis under this calculus are presented in the expressions quite familiar with biology.
3. A biological phenomenon can be analyzed in several models by simply changing the term semantic-ID. This is quite meaningful since simulation results sometimes differ among several models. The difference will give a hint for future investigation.

Table 1: Syntax of bio-calculus for molecular interaction.

| $S$ | $\begin{aligned} := & \operatorname{def} M_{\text {set }} \\ & \text { in } A \end{aligned}$ | molecular-relation-set $\delta \delta$ initial-status-set |  |
| :---: | :---: | :---: | :---: |
| $M_{\text {set }}$ | $=M_{1}, M_{2}$ | molecular-relation-set |  |
|  | $:==\quad M$ |  |  |
| M | $=T k_{v}[$ semantic $-I D]$ | molecular-relation |  |
| $T$ | $=P_{1} \Rightarrow P_{2}$ | template-relation | ID-independent-bio-syntax |
| I | $:==\quad I_{m}, I_{e}$ | initial-status-set |  |
| $I_{m}$ | $:==\quad B_{1}, B_{2}$ | initial-molecular-status-set |  |
|  | $:==\quad B$ |  |  |
| $I_{e}$ | $:==\quad E_{1}, E_{2}$ | initial-environmental-status-set |  |
|  | $:==\quad E$ |  |  |
| B | $:==x($ status $)$ | initial-molecular-status |  |
| $P$ | $:==\quad P_{1} \mid P_{2}$ | molecule-set |  |
|  |  |  |  |
| $R$ | $=x$ | molecule |  |
|  | :== $K_{1}, K_{2}$ | coefficient-set |  |
|  | $:==k_{v}$ |  |  |
| $N \in\{x, y, u, v, \ldots\}$ |  | name | ID-independent-bio-syntax |
| semantic - ID |  | semantic-ID | ID-dependent-bio-syntax |
| E |  | initial-environmental-status | ID-dependent-bio-syntax |
| status |  | molecular-status | ID-dependent-bio-syntax |
| $k_{v}$ |  | coefficient | ID-dependent-bio-syntax |

4. If the information-level ${ }^{1}$ of semanticsA is higher than that of semanticsB, a model under semantics $A$ cannot be transformed into another one under semantics $B$ by merely changing the semantic-ID. This points out requirements for better understanding of the target biological phenomenon.
5. A model can be transformed into another model under a semantics whose information-level is lower than that of the original one. This enables us to select faster semantics at the cost of accuracy simply changing semantic-ID.

## 3 First Step of Bio-calculus: Definition of Molecular Interaction

For the first step of the development of bio-calculus, we discuss molecular interaction under this calculus. Molecular interaction is one of the basis of biological phenomena. At first, we define biosyntax for molecular interaction. We then demonstrate several examples of bio-semantics. Finally, we discuss the advantages of bio-calculus, showing examples of molecular interaction.

### 3.1 Bio-syntax for molecular interaction

We define bio-syntax for molecular interaction as shown in Table 1. Bio-syntax for molecular interaction consists of two parts, molecular-relation-set and initial-status-set. Molecular-relation-set is defined between terms def and in, and describes relations among molecules (e.g. protein, mRNA,

[^0]and amino acids). Initial-status-set describes initial status of those molecules (e.g. the number of molecules) and environment around them (e.g. pH and temperature). A molecular-relation-set is a set of molecular-relation's, each of which consists of a template-relation, a semantic-ID, and a coefficient-set.

A template-relation consists of two molecule-set's, which are separated by term $\Rightarrow$. The left and right molecule-set's are interactable-molecule-set and produced-molecule-set, respectively. Each molecule-set consists of a molecule or molecule's separated by term |. A molecule represents a biological molecule (e.g. protein, mRNA, and amino acids). A template-relation describes that molecule(s) in interactable-molecule-set react and produce molecule(s) in the produced-molecule-set. The semanticID of a molecular-relation determines the semantics under which the molecular-relation is executed. The coefficient-set of a molecular-relation is a set of coefficient's (e.g. reaction factor and reaction velocity) that are necessary to execute the molecular-relation.

An initial-status-set consists of an initial-molecular-status-set and an initial-environmental-status-set. An initial-molecular-status-set is a set of initial-molecular-status's. An initial-molecular-status consists of a molecule and a status, and the status is described between parentheses ( and ). In an initial-molecular-status, the status defines the initial status (e.g. the number of molecule at the time point zero) of the molecule. An initial-environmental-status-set is a set of initial-environmental-status's, each of which represents the initial status of the environment (e.g. pH and temperature) for the molecular interaction defined by the bio-syntax.

Bio-calculus for molecular interaction is a multi-semantics system. Thus, its bio-syntax is divided into ID-dependent-bio-syntax, semantic-ID, and ID-independent-bio-syntax. Initial-status-set and coefficient-set are classified into ID-dependent-bio-syntax, whereas the others, except semantic$I D$, are ID-independent-bio-syntax.

For the design of template-rule's, we examined many expressions in biology. We also examined syntax of concurrency and communication calculi, $\pi$-calculus [6] and join-calculus [3] since molecular interaction is usually recognized as concurrency and communication. Thanks to the syntax of the template-rule's, bio-syntax comes to be quite similar to expressions in biology and join-calculus ${ }^{2}$.

### 3.2 Examples of bio-semantics for molecular interaction

We describe four bio-semantics for molecular interaction. Their mathematical backgrounds are confirmed by describing each of them in join-calculus.

### 3.2.1 Semantics1: stochastic bio-semantics (continuous version)

Under this bio-semantics, all biological molecules with the same name are represented by a molecule that holds a continuous value representing the number of the molecules. The reaction proceeds as follows: a molecular-relation is uniformly selected from executable molecular-relation's. The selected molecular-relation is then executed. The details are as follows:

1. Initialize the 'current-molecule-set' to an empty set and the 'global-time' to zero. The value of each molecule is set to the value indicated in the term status in the initial-molecular-status. Add each molecule in the initial-molecular-status-set to the 'current-molecule-set'. If the value of some molecule's is zero, remove the molecule's from the 'current-molecule-set'.
2. Uniformly choose one molecular-relation from molecular-relation's that are "executable" ${ }^{3}$.

[^1]3. Execute the selected molecular-relation. As the result, the value of each molecule in the interactable-molecule-set and that in the produced-molecule-set decreases and increases, respectively. The variation corresponds to the product of i) the values each of which represents the original value of each molecule in the interactable-molecule-set and ii) the coefficient in the molecular-relation ${ }^{4}$.
4. Increase the 'global-time'. The amount of increase is inversely proportional to the number of "executable" molecular-reaction's.
5. If the value of a molecule in the produced-molecule-set changes from zero to a positive value, which means that a molecule appears, add the molecule to the 'current-molecule-set'. On the other hand, if the value of a molecule in the interactable-molecule-set becomes zero, delete the molecule from the 'current-molecule-set'. Then return to Step 2.

### 3.2.2 Semantics2: differential equation bio-semantics

Under Semantics2, the definition of molecule is the same as that under Semantics1. All molecularrelation's are executed once at each time point. In short, under this bio-semantics, relations among molecules are executed as those defined under differential equations. The details are as follows:

1. Initialize the 'global-time' to zero. The value of each molecule is set to the value indicated in the term status in the initial-molecular-status.
2. All molecular-relation's are executed. For each molecular-relation, the value of each molecule in the interactable-molecule-set and that in the produced-molecule-set decreases and increases, respectively. The variation corresponds to the product of i) the values each of which represents the value of each molecule in the interactable-molecule-set at the current 'global-time', and ii) the coefficient in the molecular-relation. Sum up all variations of each molecule in the molecular-relation-set and vary the value of the molecule at the current 'global-time' with the summed value.
3. Increase the 'global-time' by a constant value. Then return to Step 2.

### 3.2.3 Semantics3: Michaelis-Menten bio-semantics

Under this bio-semantics, the definition of molecule is the same as that under Semantics1. All molecular-relation's are executed once at each time point. When a molecular-relation is executed, the change of the value of each molecule follows the Michaelis-Menten (M-M) equation [8]. The details are as follows:

1. The same as Step 1 in Semantics2.
2. All molecular-relation's are executed. For each molecular-relation, the value of the molecule's in the interactable-molecule-set and that in the produced-molecule-set decreases and increases, respectively. The variation follows the $\mathrm{M}-\mathrm{M}$ equation applied to i) the values each of which represents the value of each molecule in the interactable-molecule-set at the current 'global-time' and ii) the two coefficient's in the molecular-relation. Sum up all variations for each molecule in the molecular-relation-set and vary the value of the molecule at the current 'global-time' with the summed value.
3. The same as Step 3 in Semantics2.
[^2]
### 3.2.4 Semantics4: stochastic bio-semantics (discrete version)

Under Semantics4, all molecules are treated individually. The reaction proceeds as follows: From all existing molecules, uniformly choose two molecules. If the two molecules chosen can react, the reaction is executed with a predefined probability. Under Semantics1 (Section 3.2.1), all molecules with the same name are represented by a molecule. Thus, the difference between Semantics1 and Semantics4 is the treatment of each molecule. The details are as follows:

1. Initialize the 'current-molecule-set' to an empty set and the 'global-time' to zero. For each molecule, generate the same number of elements as that indicated in the term status of its initial-molecular-status. Add those elements to the 'current-molecule-set'.
2. Uniformly choose two elements from the 'current-molecule-set'.
3. If the molecule(s) representing those elements are defined to react in a molecular-relation, select the molecular-relation and go to Step 4, otherwise go to Step 6.
4. Decide whether to execute the selected molecular-relation, depending on the coefficient of the molecular-relation. The coefficient represents the probability of the execution. With this probability, go to Step 5, otherwise go to Step 6.
5. Delete the two elements selected in Step 2 from the 'current-molecule-set'. Generate an element for each of all molecule's in the produced-molecule-set and add all the generated element(s) to the 'current-molecule-set'.
6. Increase the 'global-time' by a constant value. Then return to Step 2.

### 3.3 Examples for molecular interaction

In Section 2.3, we have listed five major advantages of bio-calculus. Advantages 1 and 2 are directly related to the Req. 1-4, and Advantages 3-5 come from the multi-semantics system. In this section, we discuss those advantages of bio-calculus, showing simple examples of molecular interaction.

### 3.3.1 Advantage 1

As shown in Fig. 3, an expression of biology (Fig. 3(a)) is quite similar to a template-rule of bio-calculus. Thus the expression can be easily transported to bio-calculus (Fig. 3(b)). Then, the expression of biocalculus can be analyzed by means of computer science (Fig. 3(c)).

### 3.3.2 Advantage 2

Fig. 4(a) shows an expression in computer science for a biological phenomenon. The same biological phenomenon can be expressed in bio-calculus (Fig. 4(b)). The template-rule's of bio-syntax are easily transported to an expression of biology (Fig. 4(c)).

### 3.3.3 Advantages 3

We explain Advantage 3 using Fig. 5. Fig. 5(a) shows a bio-calculus expression of the mRNA transcription model described by Ko [4]. If the semantic-ID in this expression is semantics2, this expression is interpreted under Semantics2 and if it is semantics4, the expression is interpreted under Semantics4. Thus, an expression of bio-calculus will be evaluated under several different semantics just by exchanging semantic-ID's. It becomes an easy task to compare different semantics. If the executed results of different semantics are not the same, as in Fig. 5(b), the difference will give a hint for future investigation.

(a)

| let def |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | ProteinA $\mid$ KinaseC | $\Rightarrow$ | ProteinA_KinaseC | $k_{A \mid C}$ | semantics1 |
|  | ProteinA_KinaseC | $\Rightarrow$ | ProteinA_P $\mid$ KinaseC | $k_{A_{-} C_{-} 1}$ | semantics1 |
| , | ProteinA_KinaseC | $\Rightarrow$ | ProteinA $\mid$ KinaseC | $k_{A_{-} C_{-}}$ | semantics 1 |
| in | ProteinA $($ numA $)$, KinaseC $($ numC $)$ |  |  |  |  |

(b)


Figure 3: (a) A typical expression of biology for molecular interaction. KinaseC binds to proteinA and phosphorylate proteinA. (b) An expression of bio-calculus for the molecular interaction described in (a). (c) Executed results of the bio-calculus expression in (b). $k_{A \mid C}=0.25, k_{A_{-} C_{-} 1}=$ $0.10, k_{A_{-} C_{-} 2}=0.25, n u m A=1000, n u m C=50$
(c)

### 3.3.4 Advantage 4

The more detailed information your model contains, the more time the execution spends. To save the time, one can select other semantics under which the expression can be executed faster, at the cost of accuracy. Semantics1 treats all molecules of the same name as a molecule. Semantics4 treats those as individual elements. Thus, as shown in Fig. 6, Semantics1 is faster than Semantics 4 by $O(n)$ ( $n$ is the number of all molecules in an expression of bio-calculus).

### 3.3.5 Advantage 5

Let us assume that the information-level of SemanticsA is higher than that of some SemanticsB. If one switches a semantic-ID in a bio-calculus expression from SemanticsB to SemanticsA, bio-calculus clearly show what information is necessary for the description under SemanticsA. An example is as follows. When one tries to translate an expression of Semantics3 into that of Semantics2, each coefficient representing the M-M constant needs to be translated into two coefficients. This is because an M-M constant is the ratio of two coefficients for two simpler differential equations. Thus, when an expression under Semantics3 is translated into that under Semantics2, the values of the two coefficients - new information - are required. This fact is easily recognized in bio-calculus. Owing to this advantage, we have already obtained an insightful biological result. ${ }^{5}$

[^3]
## 4 Conclusion

In this paper, we proposed bio-calculus, an expression system that bridges a gap between biology and computer science. At first, we proposed four requirements for such expression system. We proposed the design of bio-calculus that fulfills these requirements, then pointed out five major advantages of the calculus. For the first step of the development of bio-calculus, we presented bio-syntax for molecular interaction and presented four examples of bio-semantics. Finally, we discussed the five major advantages of bio-calculus showing some examples of molecular interaction.

The distinguished feature of bio-calculus is multi-semantics. In bio-calculus, the semantics of a bio-syntax varies depending on the selected bio-semantics. This syntax-semantics relationship is the most important point of bio-calculus since a biological phenomenon should be explained by several different explanations. We discussed that many advantages of bio-calculus come from this feature. We expected that other advantages will emerge from multi-semantics system. We believe that multisemantics system can also be applied for expressing other phenomena existing in the real world.

Currently, we are developing a system that automatically generate the cell lineage from 4 D microscope images of a C. elegans embryo [9]. We are planning to apply bio-calculus to the system and further extend the system in order to realize automatic gene interaction analysis using the enormous data produced by genomics.

For the first step of the development of bio-calculus, we defined the bio-syntax for molecular interaction since molecular interaction is one of the bases of biological phenomena. We are planning to extend the calculus for sub-cellular localization and cell-cell interaction to apply this calculus for the development of multi-cellular organisms.

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$$
\begin{aligned}
& \frac{d}{d t}[\text { cyclin }]=k_{1}-[\text { cyclin }]\left(V_{2}^{\prime}[A P C]+V_{2}^{\prime \prime}\left[A P C^{*}\right]+k_{3}[c d c 2]\right) \\
& \frac{d}{d t}\left[c d c 2_{-} c y c l i n\right]=k_{3}[c y c l i n][c d c 2]+k_{\text {cakr }}\left[c d c 2_{-} p_{\_} c y c l i n\right]+V_{25}^{\prime}[c d c 25]\left[P \_c d c 2_{-} C y c l i n\right]+ \\
& V_{25}^{\prime \prime}[c d c 25]\left[P \_c d c 2 \_c y c l i n\right]+k_{i r}\left[C K I \_c d c 2 \_C y c l i n\right] \\
& -\left[c d c 2 \_c y c l i n\right]\left\{\left(V_{2}^{\prime}[A P C]+V_{2}^{\prime \prime}\left[A P C^{*}\right]\right)+k_{c a k}+\right. \\
& \left.\left(V_{\text {wee }}^{\prime}\left[W e e \_P\right]+V_{\text {wee }}^{\prime \prime}[W e e]\right)+k_{i}[c d c 2]\right\}
\end{aligned}
$$

(a)

| let def |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | amino - acid | $\Rightarrow$ | cyclin | $k 1$ | semantic - ID |
| , | cyclin $\mid$ APC* | $\Rightarrow$ | amino - acid $\mid$ APC ${ }^{*}$ | $V_{2}^{\prime \prime}$ | semantic - ID |
| , | cdc2_p_cyclin $\mid$ cakr | $\Rightarrow$ | cdc2_cyclin $\mid$ cakr | $k_{\text {cakr }}$ | semantic - ID |
| , | $p \_c d c 2 \_c y c l i n \mid c d c 25 \_P$ | $\Rightarrow$ | cdc2_cyclin $\mid c d c 25 \_P$ | $V_{25}^{\prime \prime}$ | semantic - ID |
| , | cdc2_cyclin $\mid$ APC | $\Rightarrow$ | amino - acid $\mid$ APC | $V_{2}^{\prime}$ | semantic - ID |
| , | cdc2_cyclin \|cak | $\Rightarrow$ | cdc2_p_cyclin \|cak | $k_{\text {cak }}$ | semantic - ID |
| , | cdc2_cyclin $\mid$ Wee | $\Rightarrow$ | P_cdc2_cyclin $\mid W e e$ | $V_{\text {wee }}^{\prime \prime}$ | semantic - ID |
| , | cyclin $\mid$ APC | $\Rightarrow$ | amino - acid $\mid$ APC | $V_{2}^{\prime}$ | semantic - ID |
| , | cyclin $\mid$ cdc 2 | $\Rightarrow$ | cdc2_cyclin | $k_{3}$ | semantic - ID |
| , | P_cdc2_cyclin $\mid$ cdc 25 | $\Rightarrow$ | cdc2_cyclin $\mid$ cdc 25 | $V_{25}^{\prime}$ | semantic - ID |
| , | CKI_cdc2_cyclin | $\Rightarrow$ | CKI $\mid$ cdc2_cyclin | $k_{i r}$ | semantic - ID |
| , | cdc2_cyclin $\mid$ APC* | $\Rightarrow$ | cdc2_cyclin $\mid A P C^{*}$ | $V_{2}^{\prime \prime}$ | semantic - ID |
| , | cdc2_cyclin $\mid W e e P$ | $\Rightarrow$ | P_cdc2_cyclin $\mid W e e P$ | $V_{\text {wee }}^{\prime}$ | semantic - ID |
| , | cdc2_cyclin \| $C K I$ | $\Rightarrow$ | CKI_cdc2_cyclin | $k_{i}$ | semantic - ID |
| , | ... |  |  |  |  |

(b)


Figure 4: (a) Differential equations demonstrated in Novak et al. [5]. They used the differential equations for modeling the cell-cycle in Xenopus embryo. cyclin, $A P C, A P C^{*}$, $c d c 2 \_c y c l i n, c d c 2, W e e, W e e_{-} P, \ldots$ means proteins and $k_{1}$, $k_{c a k}, k_{c a k r}, V_{25}^{\prime}, V_{25}^{\prime \prime}, V_{2}^{\prime}, V_{2}^{\prime \prime}, V_{w e e}^{\prime}, V_{w e e}^{\prime \prime}, k_{i} \ldots$ means coefficients. Only two equations are shown for saving the space. (b) A bio-calculus expression corresponding to the above simulation model. (a). Only several molecular-relations are shown. (c) A typical expression of biology representing several molecular reactions expressed in the above bio-calculus expression (b).
(c)

| let def |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $D N A \mid$ unbound | $\Rightarrow$ | bound $\mid m R N A$ | $P 1$ |
|  | semantic $-I D$ |  |  |  |
|  | N $A \mid$ unbound | $\Rightarrow$ | $D N A \mid$ unbound | $1-P 1$ |
| semantic $-I D$ |  |  |  |  |
|  | $\Rightarrow$ bound | $\Rightarrow$ | $D N A\|Q\|$ unbound | $P 2$ |
| polII $\mid$ bound | $\Rightarrow$ | $\operatorname{polII} \mid$ bound $\mid m R N A$ | $1-P 2$ | semantic $-I D$ |
| in $D N A(1), \operatorname{polII}(1), Q(1), \operatorname{polII}(1)$, unbound $(1)$ |  |  |  |  |
|  |  |  |  |  |

(a)

(b)


Figure 5: (a) An expression of bio-calculus representing the model of mRNA transcription described by Ko [4]. $\quad(P 1=0.90, P 2=0.0005$, stepnumber $=$ $51, m R N$ A transcription number for each step $=1)$. $\quad(b)$ The execution results of the bio-calculus description in (a). The solid line is the result of the execution under semantics2. Each of other lines represents each result of five independent executions under semantics4.

Figure 6: The ratio of the execution time under semantics1 to that under semantics4 for the same bio-calculus expression. The x -axis is the number of all molecules in an expression of bio-calculus. The y-axis is a relative ratio of execution time ( $\left.\frac{\text { execution time under semantics } 1}{\text { execution time under semantics } 4}\right)$.


[^0]:    ${ }^{1}$ Information-level of semantics $\mathrm{X}\left(\inf o_{x}\right)$ is the amount of information (such as parameters) that is necessary for the expression of bio-calculus under semantics X. A relationship between two information-level is defined in partial order.

[^1]:    ${ }^{2}$ Join-calculus and $\pi$-calculus has the same expressive power [3]. The syntax of join-calculus based on the chemical metaphor [2]. Thus, bio-syntax was designed to be similar to the syntax of join-calculus.
    ${ }^{3}$ We say that a molecular-reaction is "executable" if all molecules in the interactable-molecule-set exists in 'current-molecule-set'.

[^2]:    ${ }^{4}$ We do not use term "coefficient-set" but term "coefficient", because the semantics uses only one coefficient.

[^3]:    ${ }^{5}$ We have obtained an insightful biological result, by rewriting the paper [5]. This paper models the cell cycle of Xenopus embryo with differential equations which contain Michaelis-Mentens equation, which we translated to semantics1.

